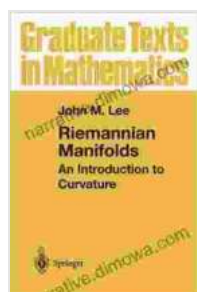


An Introduction to Curvature: A Journey into the Heart of Differential Geometry

Unveiling the Secrets of Curved Spaces

Embark on an intellectual odyssey into the realm of curvature, a fundamental concept that pervades the study of surfaces, manifolds, and the intricate fabric of our universe. 'An to Curvature: Graduate Texts in Mathematics 176' is your trusted guide on this captivating journey, offering an accessible yet rigorous exploration of curvature's multifaceted nature.



Riemannian Manifolds: An Introduction to Curvature (Graduate Texts in Mathematics Book 176) by John M. Lee

★★★★☆ 4.5 out of 5

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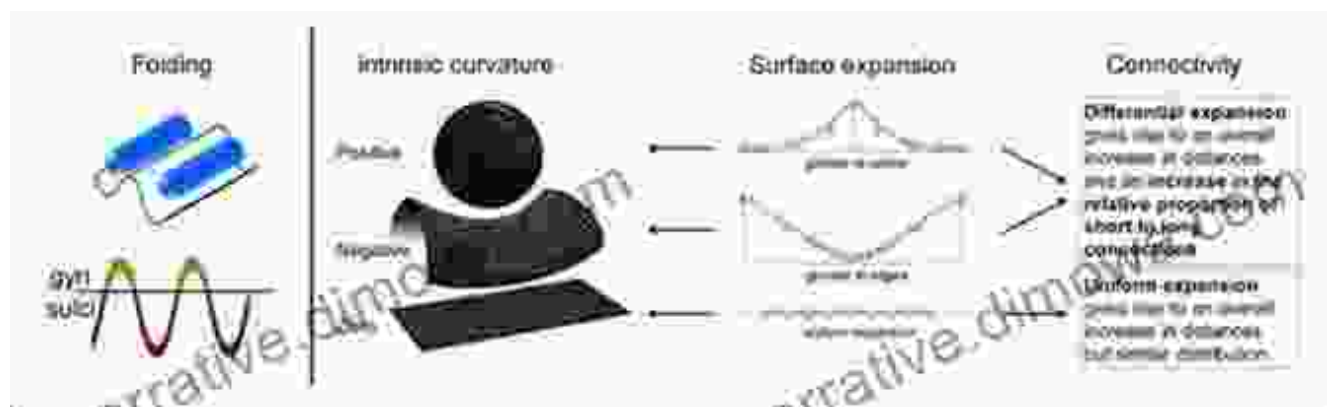
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A Tapestry of Curvature Concepts

Within the pages of this comprehensive tome, you'll encounter a panorama of curvature concepts, each holding a unique key to unraveling the mysteries of curved spaces. From Gaussian curvature, which measures the intrinsic curvature of surfaces, to Ricci curvature and sectional curvature,

which illuminate the curvature properties of Riemannian manifolds, the book delves into the intricacies of these fundamental notions.



Bridging Theory and Application

'An to Curvature' seamlessly weaves together theoretical foundations and practical applications, providing a comprehensive understanding of this multifaceted concept. Through carefully crafted examples and insightful discussions, the book highlights the profound implications of curvature in fields ranging from differential geometry and topology to general relativity and cosmology.

On the Curvature of Pattern Transformation Manifolds: Numerical Estimation and Applications

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Abstract

This paper addresses the numerical estimation of the principal curvatures of pattern transformation manifolds. When a visual pattern undergoes a geometric transformation, it traces a (possibly high-dimensional) manifold, which is usually called the transformation manifold. The manifold curvature is an important property characterizing the manifold geometry, with several applications in manifold learning. We propose an efficient numerical algorithm for estimating the principal curvatures at a given point on the transformation manifold.



Figure 1: The parameter space H provides a parametrization of the transformation manifold M .

Introduction

We study the problem of estimating quantitatively the principal curvatures of pattern transformation manifolds, which are formed by points in \mathbb{R}^d under geometric transformations (e.g., rotations, scaling, and so on). The principal curvatures are the d largest eigenvalues of an appropriate Hessian matrix, which captures the local geometry of the manifold near a given point. For example, the curvature can be a valuable tool for manifold discrimination towards transformation-invariant pattern recognition. In particular, the works (Pohlmann and Hengler, Kulis, Susskind, Parillo, and Frossard) introduce geometrically transformed versions of the data samples, known as virtual samples, to make graph-based classification methods robust to geometric transformations. However, one faces in this case the problem of constructing the virtual samples or, equivalently, the problem of manifold discrimination.

In the context of manifold learning, it has been shown in (Frossard and Wakin) that the manifold curvature number, which is closely related to the visual pattern, is an important factor towards characterizing the manifold geometry, that are needed for obtaining an isometric embedding of the manifold under random projections. Therefore, the curvature estimation has received increasing attention in manifold learning. We propose here an efficient and numerically stable algorithm for estimating the principal curvatures at a given point on the transformation manifold.

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Numerical estimation of the curvature

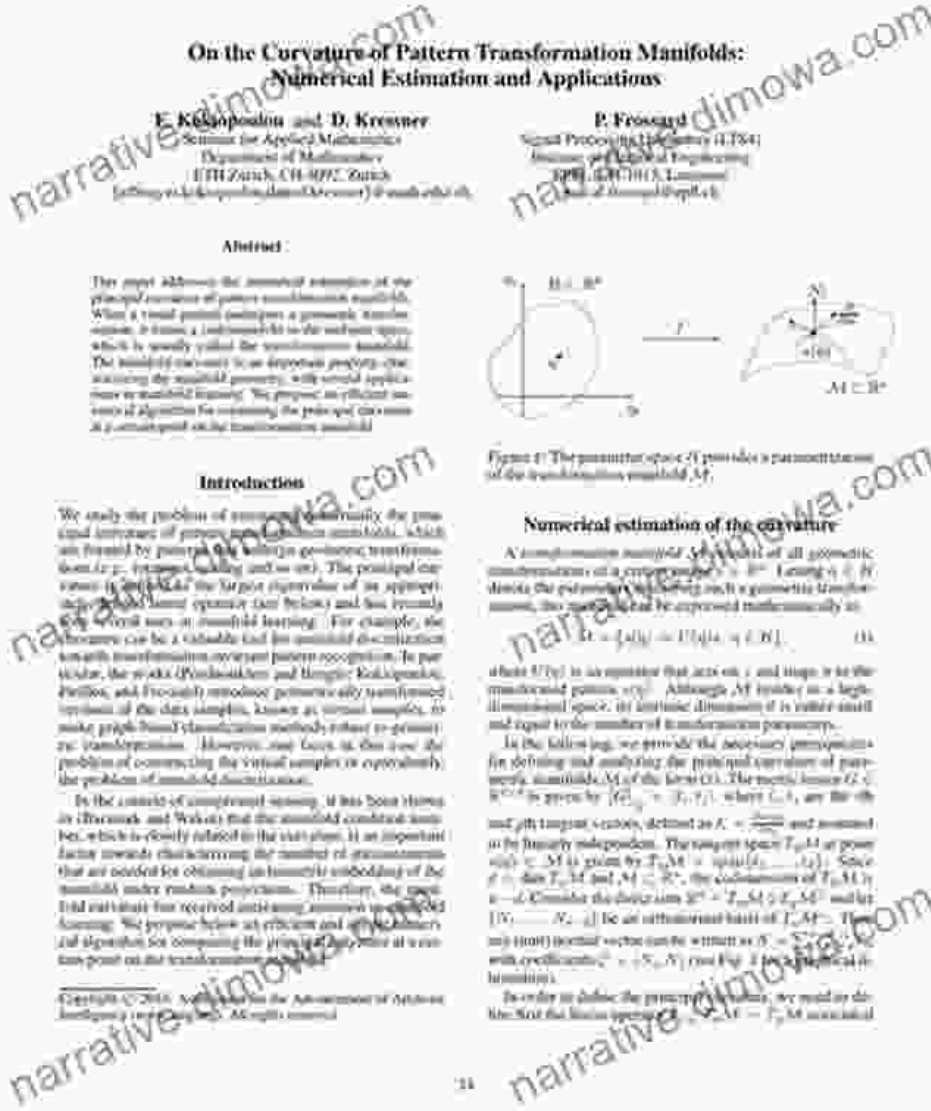
A transformation manifold M is the set of all geometric transformations of a given image $x \in \mathbb{R}^d$. Letting $q \in H$ denote the parameter describing such a geometric transformation, the manifold can be expressed mathematically as

$$M = \{T(q) \mid q \in H\}, \quad (1)$$

where $T(q)$ is an operator that acts on x and maps it to the transformed pattern $x(q)$. Although M resides in a high-dimensional space, its intrinsic dimension d is rather small and equal to the number of transformation parameters.

In the following, we provide the necessary preliminaries for defining and analyzing the principal curvatures of parameter manifolds M of the form (1). The manifold $M \subset \mathbb{R}^d$ is given by $\{T(q) \mid q \in H\}$, where $H \subset \mathbb{R}^n$ and T maps q to $T(q)$ and assumed to be linearly independent. The tangent space $T_p M$ at point $p \in M$ is given by $T_p M = \text{span}\{v_1, \dots, v_d\}$. Since $p \in M$, due to (1) and $M \subset \mathbb{R}^d$, the codimension of $T_p M$ is $d - d$. Consider the domain $\mathbb{R}^d = T_p M \oplus T_p M^\perp$ and let $\{v_1, \dots, v_d, v_{d+1}, \dots, v_d\}$ be an orthonormal basis of $T_p M^\perp$. The d (non) normal vectors can be written as $v_i = \sum_{j=1}^d \alpha_{ij} v_j$ with coefficients $\alpha_{ij} \in \mathbb{R}$ (see Fig. 1 for a graphical illustration).

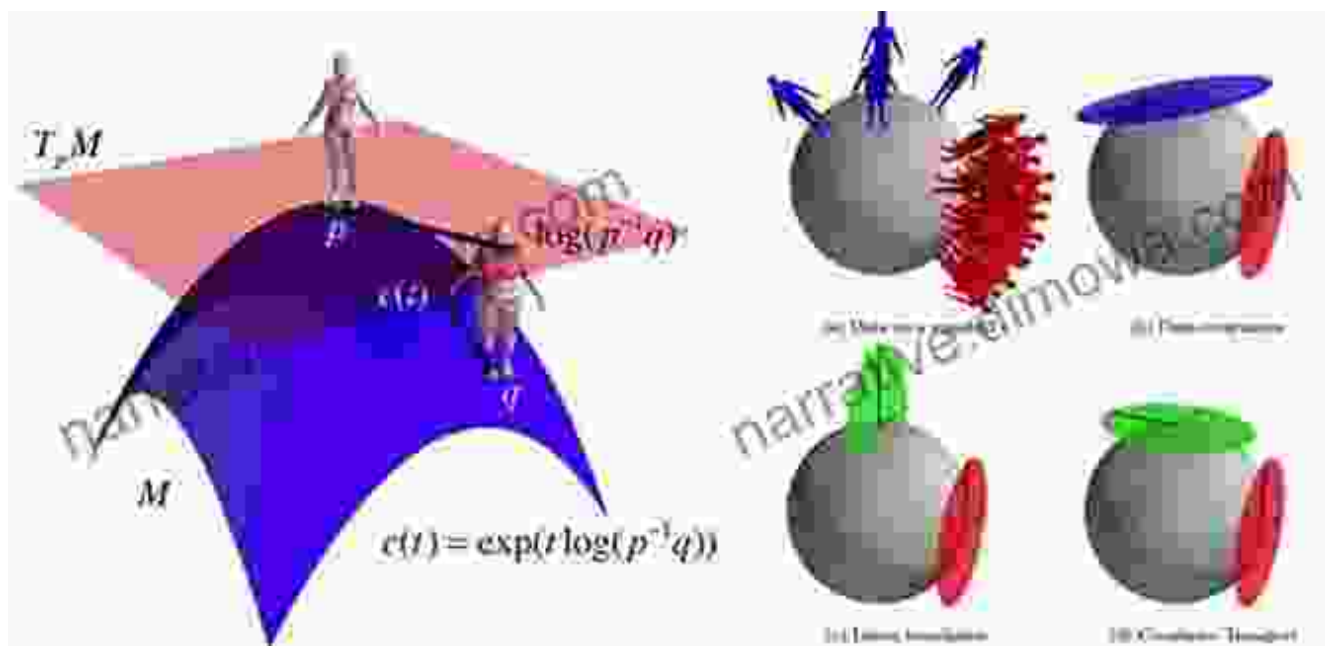
In order to define the principal curvatures, we need to define the SOT of the linear operator $L = T_p M$ associated



Delving into Differential Geometry

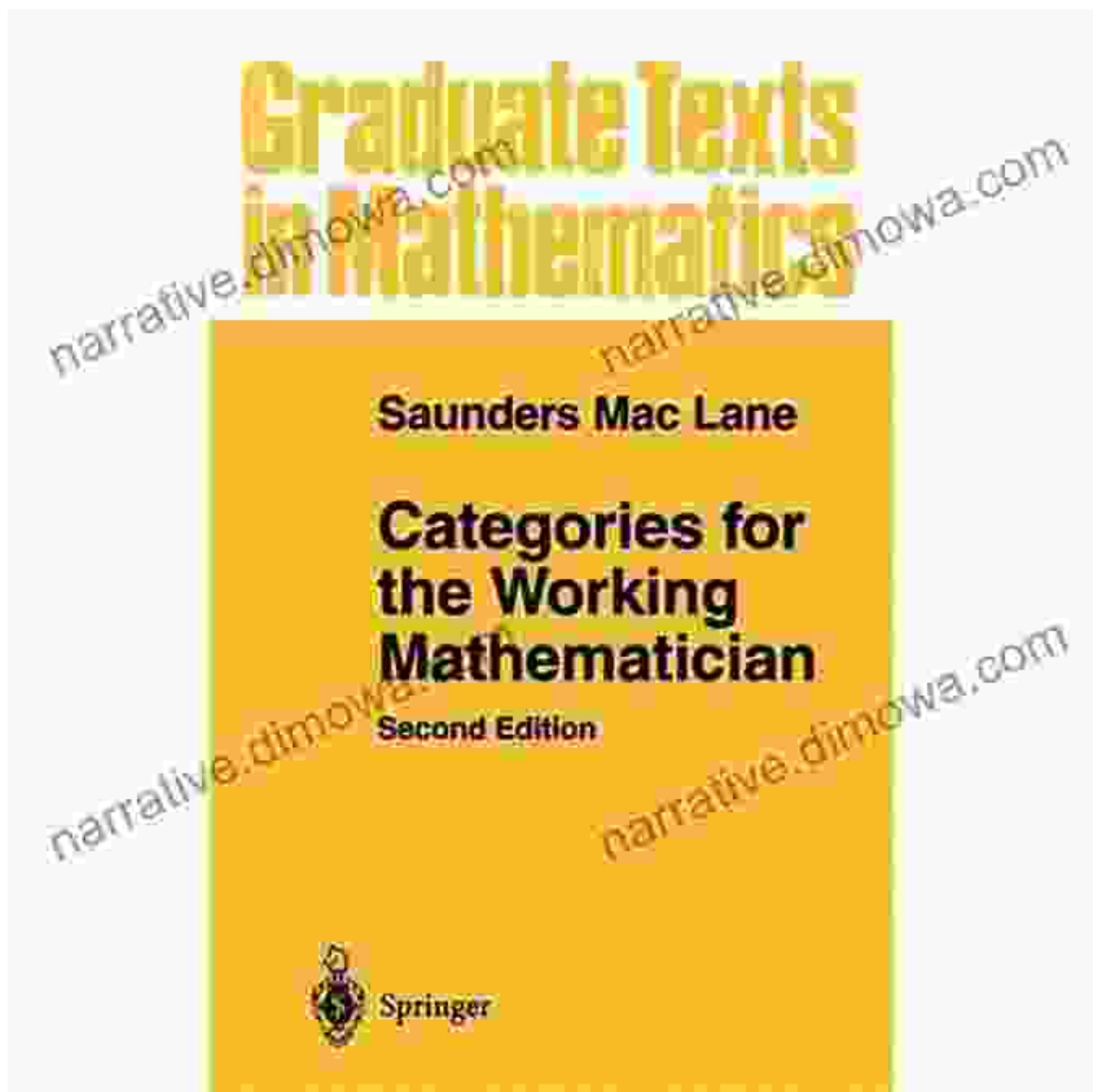
As you delve deeper into the book's chapters, you'll gain a profound understanding of differential geometry, the branch of mathematics that explores the geometry of smooth manifolds—curved surfaces that can be locally approximated by Euclidean space. Curvature serves as a central

pillar in differential geometry, providing essential insights into the intrinsic properties of these fascinating geometric objects.



Empowering Your Mathematical Journey

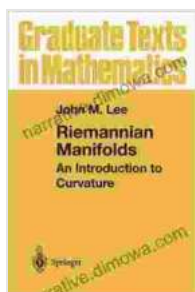
'An to Curvature' is an invaluable resource for graduate students, researchers, and anyone seeking to expand their knowledge of curvature and its profound implications. Whether you're a seasoned mathematician or an aspiring explorer of curved spaces, this book will empower you with the theoretical foundations and practical tools to navigate the complexities of this captivating mathematical realm.



Free Download Your Copy Today

Don't miss out on this opportunity to embark on an enlightening journey into the captivating world of curvature. Free Download your copy of 'An to Curvature: Graduate Texts in Mathematics 176' today and unlock the secrets of curved spaces. Your intellectual odyssey awaits!

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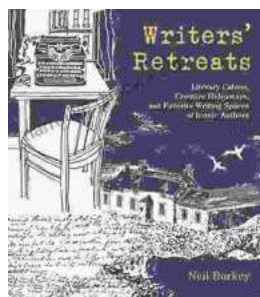
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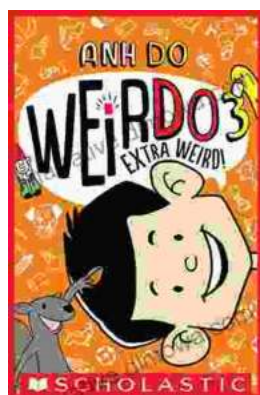
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